

1987 (10)

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(a)  $2x(1+y) \frac{dx}{dy} = 8+x^2$

$\Rightarrow \int \frac{2x}{x^2+8} dx = \int \frac{1}{1+y} dy + A$  [5]

$\Rightarrow \ln(x^2+8) = \ln(1+y) + A$  [5]

//  $x=2, y=3 \Rightarrow$

$\Rightarrow \ln(2^2+8) = \ln(4) + A$

$\Rightarrow \ln(12) = \ln(4) + A$  [5]

$\Rightarrow \ln(12) - \ln(4) = A$

$\Rightarrow \ln\left(\frac{12}{4}\right) = A$

$\Rightarrow \ln(3) = A$  //

$\Rightarrow \ln(x^2+8) = \ln(1+y) + \ln(3)$

$\Rightarrow \ln(x^2+8) = \ln[3(1+y)]$

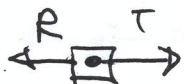
$\Rightarrow x^2+8 = 3(1+y)$

$\Rightarrow x^2 = 3y - 5$

$\Rightarrow x = \sqrt{3y-5}$  [5]

(b) on the level: we need to calculate the power output of the train as this is we are told the power output on the slope also.

Force:



Speed = Constant =  $16 \text{ m/s}$

$R = 60 \text{ N}$  per tonne

NII  $\Rightarrow T - R = 0$  [a=0]

$\Rightarrow T = R$

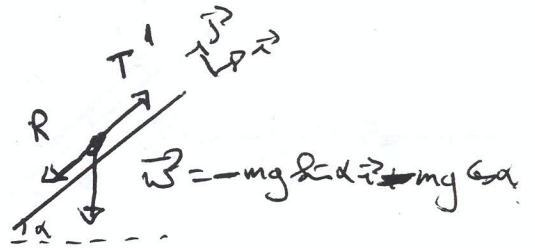
$\Rightarrow T = \frac{60 \text{ m}}{1000}$  [Careful of units]

Power output =  $T \cdot v$   
 $= \left(\frac{60 \text{ m}}{1000}\right) 16$   
 $= \frac{96 \text{ m}}{100}$

on the slope:

Power output of engine =  $\frac{96 \text{ m}}{100}$

Force:



Along plane:

NII  $\Rightarrow ma = \Sigma F$

$\Rightarrow \frac{m dv}{dt} = T' - mg \sin \alpha - R$  [5]

BUT: Power =  $\frac{96 \text{ m}}{100}$

$\Rightarrow T' v = \frac{96 \text{ m}}{100} \Rightarrow T' = \frac{96 \text{ m}}{100 v}$  [5]

\*  $\Rightarrow$

$\frac{m dv}{dt} = \frac{96 \text{ m}}{100 v} - \frac{mg}{48} - \frac{60 \text{ m}}{1000}$  [5]

$\Rightarrow \frac{dv}{dt} = \frac{96}{100 v} - \frac{9 \cdot 8}{48} - \frac{6}{100}$

$\Rightarrow \frac{dv}{dt} = \frac{96}{100 v} - \frac{1}{10} - \frac{6}{100}$

$\Rightarrow \frac{dv}{dt} = \frac{96}{100 v} - \frac{16}{100}$

$\Rightarrow \frac{dv}{dt} = \frac{16}{100} \left(\frac{6}{v} - 1\right)$

$\Rightarrow \frac{dv}{dt} = \frac{4}{25} \left(\frac{6-v}{v}\right)$  [5]

$\Rightarrow \int_{16}^{12} \frac{v dv}{6-v} = \int_0^T \frac{4}{25} dt$

$\Rightarrow \int_{16}^{12} \frac{v}{v-6} dv = \int_0^T \frac{4}{25} dt$

$\int \frac{v}{v-6} dv = \int \frac{v+6-6}{v-6} dv$

Let  $u = v-6$   
 $\Rightarrow u+6 = v$   
 $du = dv$

$\Rightarrow \int \frac{v}{v-6} dv = \int \frac{u+6}{u} du$

$= \int 1 + \frac{6}{u} du$

$= [u + 6 \ln u]$  [5]

$= [(v-6) + 6 \ln(v-6)]$  //